

# An algorithm for optimal management of aggregated HVAC power demand using smart thermostats

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## HIGHLIGHTS

- An algorithm for optimal management of aggregated HVAC power is presented.
- The HVAC control problem is formulated as a jobs scheduling problem.
- The algorithm is analytically proven to be optimal.
- Simulation results show an improvement in HVAC demand reduction by 130% over a traditional approach.
- Demand restrike is limited to pre-DR-event level.

## ARTICLE INFO

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## ABSTRACT

This paper presents an algorithm for optimal management of aggregated power demand of a group of heating, ventilating and air-conditioning (HVAC) units. The algorithm provides an advanced direct load control mechanism for HVACs that leverages the availability of smart thermostats, which are remotely programmable and controllable. The paper provides a theoretical basis and an optimal solution to the problem of cycling a large number of HVAC units while respecting customer-chosen temperature limits for the purpose of maximum load reduction. The problem is presented in a new light by transforming it into a job scheduling problem and is solved using a combination of a novel greedy algorithm and a binary search algorithm. By leveraging widespread availability of smart internet-based (also referred to as IoT-based) thermostats in today's environment, the proposed approach can be readily applied to residential buildings without additional electrical/IT infrastructure changes.

## 1. Introduction

Demand side management is an alternative to achieving energy balance in the electric grid by means of altering electrical demands to suit available supply. Historically, demand side management has been used for long term energy balance through energy efficient appliances, financial incentives, consumer education and government regulation [1]. It has evolved to incorporate load profile management through energy audits, direct load control and subsequently real time pricing [2]. Demand response (DR) is one method of demand side management where end-use electricity consumption changes in response to changes in electricity price or to alleviate system stress condition [3]. Surveys on various DR methods are available in [4–6] and real-world applications can be found in [7]. One of the most popular DR methods for managing aggregated power of a group of customers is direct load control (DLC) [8] in the residential sector [9], where a utility remotely turns off

electrical equipment at customer premises, e.g., HVACs, during the time of system stress, disregarding customer comfort.

In the literature on smart DLC for HVAC, HVACs are crudely divided into groups based on their comfort requirements and building thermal characteristics, then an empirical fuzzy rule [10] or a predictive control method [11] is applied to control the duty-cycle for each group. These methods employed, however, neither explicitly take care of customer temperature preference nor achieve optimal load reduction. Authors in [12] use peculiar chilled water thermal storage capacity to offer DLC in commercial buildings, which cannot be generalized for the residential settings. In [13] adaptive control based DLC is explored where a power reduction requirement is converted to set-point change requirement to feed into a classical thermostat based controller. The study shows a simulation result where the power has been successfully limited to 90% of the original peak but whether that is the maximum reduction attainable is unknown. Similarly arbitrary change of temperature set-

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points are shown to achieve some arbitrary reduction of power consumption during a critical period in [14]. In [15] double auction based transactive control is used to set-up a capacity market to limit the aggregated HVAC power. However, the power limit imposed is arbitrarily chosen and the transactive control used may not be optimal to control the population of HVACs. In [16] dynamic price based transactive control is studied for its efficacy in reducing the peak. But the price signal used is arbitrarily chosen and the reduction attained is far from optimal. Our previous work on transactive control [17] shows that a simple price signal based transactive control can reduce load during a high price period but creates a restrike after the event.

There are also a substantial number of research work on optimal DR, focusing on task scheduling problems. Authors in [17–19] solve an optimization problem to calculate the optimal schedule for each appliance that incurs minimum cost. This method only addresses scenarios where buildings are concerned about their cost minimization but the presence of other households trying to do the same thing has no effect on the solution. Authors in [20] indirectly address this problem by setting the upstream price as a function of the aggregated power, but it is still based on a primitive task-scheduling problem and as such does not take into account the fine thermal dynamics of HVACs. Similarly, authors in [21–23] address aggregated appliance control through optimal scheduling, but peculiar HVAC dynamics is ignored. In our previous work [24], DR at a single house is achieved by scheduling loads according to their priority and comfort requirement but the application of the algorithm for aggregated cases remains unexplored. In [25], HVAC thermal dynamics are incorporated but individual comfort constraints and the aggregated dynamics are ignored. In [26], HVAC thermal dynamics is taken into account and optimization for cost reduction based on dynamic price is performed but aggregated dynamics is disregarded.

There are also research work that specially targets aggregated control of HVAC or thermostatically controlled loads (TCL). In [27], a probabilistic aggregate model of a collective HVAC system is used for demand response but the aggregate power frequently exceeds the reference and also temperatures of some buildings exceed customer preference limits. In [27–29], an aggregated model is developed for HVACs and their potential for load following is explored. HVACs are controlled based on priority established by calculating temperature distance from the boundary. Authors in [30] develop a similar approach for water heater control. Although the control methods used in these papers are intuitive their optimality is not proven. The research work in [31] converts HVACs into generic second order continuous time TCLs and creates an aggregated battery model to study the collective behavior on average; but the aggregate model cannot take into account instantaneous co-incidence event of different HVACs. In [32], an iterative demand bidding is used to arrive at an optimal schedule of customers so that their collective utility is maximized—but there is no direct control on the amount of power reduction and might take long time for the iterative bidding to stabilize and deliver HVAC control schedules.

In more recent works, an aggregated model of AC has been developed and an optimal DR based on dynamic programming that respects comfort constraints has been proposed in [33]. Similarly, aggregated control of residential HVAC for peak load shaving has been explored in [34] and optimal DR using MPC approach can be found in [35–37]. In [38], authors present mechanism for optimal scheduling of smart appliances in the context of a smart community for the purpose of peak load reduction. Transactive control based DR is explored in [39]. A generic modelling technique also applicable to HVAC for fast-acting DR can be found in [40]. However, all of these research is concerned with optimal DR in presence of an upstream price signal or by creating various pricing scheme, which is not applicable to the problem we are exploring. In [41], aggregated control of HVAC for frequency regulation using a sliding-window based control is explored while their control for load balancing service is explored in [42].

In the area of DLC for HVACs for peak shaving, recent work close to

ours can be found in [43]. While authors in [43] minimize user comfort violations by doing a fair allocation of the comfort violations to reduce the peak load by an arbitrarily set amount, this work aims at meeting the user comfort requirement (that the user agreed to in the contract), and achieving maximum attainable load reduction. Hence the novelty and contribution of this work is to find the maximum possible load reduction, and show a method to optimally achieve it, backed by an analytical proof.

We propose a novel control algorithm for a population of HVAC units that uses a combination of a novel greedy algorithm and a binary search algorithm to keep the aggregated demand (kW) at the lowest possible level during a control duration. The proposed approach explicitly considers comfort constraints (in terms of upper and lower temperature limits) of individual customers by maintaining indoor temperatures within their preference bounds. Although there is abundant research that considers HVAC thermal dynamics, user comfort constraints and aggregation effects for HVAC control, as per our knowledge, this approach in the context of finding the maximum load reduction potential given a DR period is new. The proposed greedy algorithm for HVAC control is simple, intuitive and highly efficient. This paper specifically analyzes HVAC control dynamics, presents an intuitive problem formulation for optimal load control, and analytically proves the solution optimality. This contributes to theoretical clarity on the problem of aggregated HVAC control. Although the work presented here can be easily generalized to any TCL, this paper focuses on HVAC control as it is the most popular one in today's environment, and there is a widespread adoption of smart WiFi thermostats. No such smart thermostats are widely available for controlling other TCLs like water heaters or refrigerators. This means large scale implementation of the proposed algorithm for HVAC control is possible without any additional infrastructure and without the installation of A/C cycling switch that is being done today for HVAC DLC. Although applicable for both heating and cooling, for brevity, the rest of the paper focuses on the cooling mode of operation.

The contributions of the paper can be summarized as following:

- (1) The paper presents a novel linear-time algorithm to find the maximum load reduction potential for an aggregation of houses such that their comfort requirements are not violated.
- (2) Associated algorithm to optimally control the HVACs in those houses such that the aggregated power is kept at the minimum value while respecting the comfort requirement is also presented.
- (3) The problem of controlling the HVACs is transformed into an intuitive form of Job scheduling problem which provides theoretical clarity to the problem.
- (4) The optimality of the algorithm is analytically proven.
- (5) Optimal control of HVAC in response to DR signals presents a novel and important work in the field of intelligent building energy management system.

## 2. Framework and problem formulation

Let us consider an aggregation of  $N$  residential customers who have signed up with a DR aggregator, like EnerNoc, which can perform collective control of these customers to provide load reduction DR service to the utility or to bid this demand reduction potential into a capacity market [44]. It should not be hard to find willing participants for the proposed program since there are already customers who are participating in traditional HVAC DLC programs where they agree to let the utility remotely turn off their HVAC for a fixed duration of time (without regards for the indoor temperature) in exchange for some rebates. At minimum those customers should be willing to participate in the proposed form of DR which ensures that the temperature will be kept within a pre-agreed comfort bounds. The overall framework is shown in Fig. 1, where there is one aggregator and  $N$  houses, each with an IoT-based thermostat. There is also a controller dedicated for each

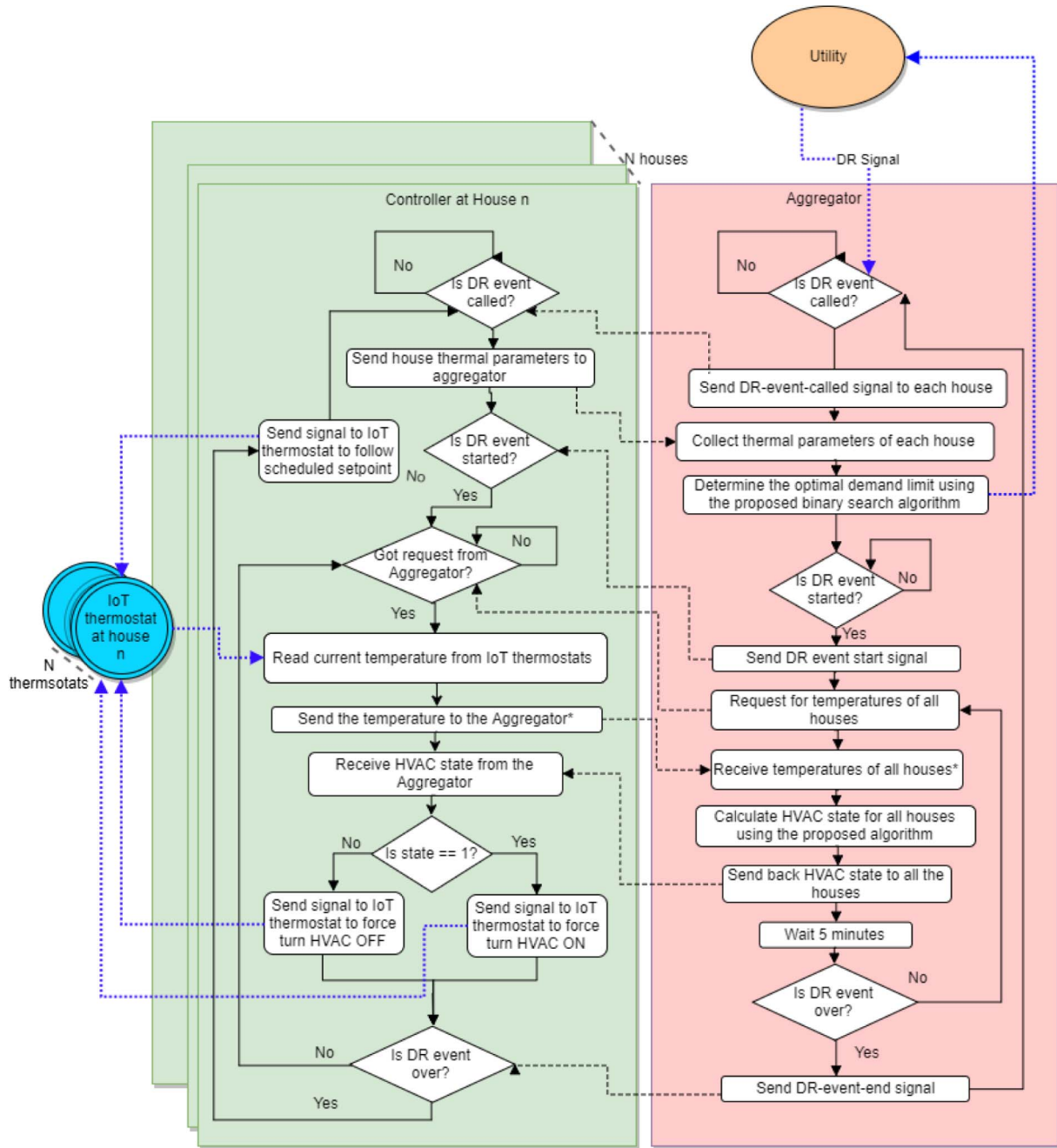


Fig. 1. Framework for optimal DR.

house, which logically belongs to the house, but can be implemented as an independent agent at the aggregator.

When the aggregator gets a call for DR service, it sends a DR-event called signal to the controllers at each house and requests for house thermal parameters and customer's preferred temperature boundaries. The controller can estimate house thermal parameters based on known properties (such as, floor area, insulation types, etc.) or from house thermal response using maximum likelihood based estimator [45]. The aggregator then determines optimal demand limit (kW) using the proposed binary search algorithm (discussed in Section 3.1). It can now bid this DR service into the capacity market if required or report this value to the utility.

When the time for DR event comes, the aggregator sends a signal to each controller and the controller goes into DR mode. At the beginning of each subsequent control interval throughout the DR event, the

aggregator requests for the internal temperature of each house. The controller communicates with its respective IoT-based thermostat to get the current temperature and calculates the time for the temperature to hit the comfort limit. It then forwards this information to the aggregator.

After receiving the information from all houses, the aggregator computes the optimal HVAC state for the current control period for each house using the proposed greedy algorithm (discussed in Sections 3.2 and 3.3) which it sends back to each controller for implementation. The controller then keeps the HVAC state OFF or ON during the current control period based on the state assigned. Usually, modern IoT-based thermostats do not provide a mechanism for direct compressor control to enforce the ON or OFF state, but this can be achieved fairly easily by remotely increasing/decreasing their set-points by at least 5 deg from their current temperature readings, forcing them to turn ON or OFF

instantly. 5 deg is deemed to be sufficient margin to overcome the deadband so as to maintain the state until the next control period. The whole process is then repeated until the end of the DR event, at which case, the controller restores the regular scheduled set-point on the IoT-based thermostat, allowing it to follow the normal deadband based set-point following mode.

For load reduction demand response purpose, such as the New York ISO special case resources program [46] or NYSEG's Distribution Load Relief Program (DLRP) [47], it is required to reduce the load by a certain value to qualify for the incentives. The problem of finding the optimal demand reduction (kW), so that the aggregated demand during a duration  $T$  from  $t_{start}$  to  $t_{end}$  is kept at the minimum, while respecting comfort constraints of individual customers can be mathematically expressed as shown in (1).

$\min D_L$

Subject to:

$$\theta_{lower_n} \leq T_{A_n}^{t_k} \leq \theta_{upper_n} \quad \forall n, \forall k$$

$$\sum_{n=1}^N P_{HVAC_n} * U_n^{t_k} \leq D_L \quad \forall k$$

$$T_{A_n}^{t_{k+1}} = f(T_{A_n}^{t_k}, T_{M_n}^{t_k}, C_{T_k}^{t_k}, T_o^{t_k}, \Delta t, U_n^{t_k}) \quad (1)$$

where

- $D_L$ : the demand limit (kW)
- $t_k$ : time step. Duration  $T$  is divided into a series of time steps  $t_{start} \leq t_k < t_{end}$
- $T_{A_n}^{t_k}$ : indoor air temperature of house  $n$  at time step  $t_k$  (°F)
- $\theta_{lower_n}$ : lower bound of acceptable temperature - house  $n$  (°F)
- $\theta_{upper_n}$ : upper bound of acceptable temperature - house  $n$  (°F)
- $P_{HVAC_n}$ : rated power of HVAC at house  $n$  (kW)
- $U_n^{t_k}$ : HVAC state (1 = ON/0 = off) for house  $n$  at time step  $t_k$
- $T_{A_n}^{t_{k+1}}$ : air temperature in the next time step - house  $n$  (°F)
- $f$ : a function that models second order thermal dynamics of a house and expresses indoor air temperature in the next time step based on the following parameters
- $T_{A_n}^{t_k}$ : air temperature in at time step  $t_k$  (°F)
- $T_{M_n}^{t_k}$ : building mass temperature at time step  $t_k$  (°F)
- $C_{T_k}^{t_k}$ : house thermal parameters (e.g., insulation, heat gains and thermal capacity) at time step  $t_k$
- $T_o^{t_k}$ : outdoor air temperature at time step  $t_k$  (°F)
- $\Delta t$ : the interval between two time steps

The second order equivalent thermal parameter (ETP) model [48,49] as shown in Fig. 2 is used to determine function  $f$ .

Where

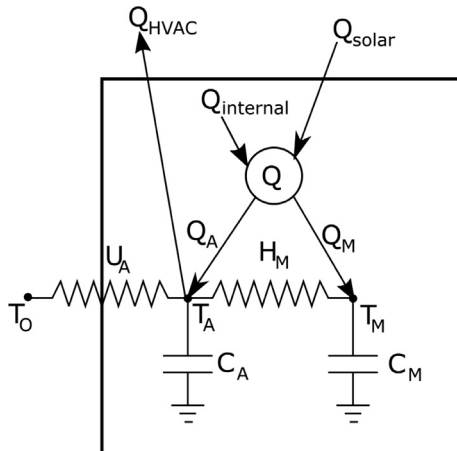


Fig. 2. ETP model for a house.

- $Q_A$ : fraction of heat injected into indoor air by internal sources ( $Q_{internal}$ ), and solar radiation ( $Q_{solar}$ ) (Btu/h)
  - $Q_M$ : the other fraction of heat injected into building mass by  $Q_{internal}$  and  $Q_{solar}$  (Btu/h)
  - $Q_{HVAC}$ : Heat removed from indoor air by the HVAC (Btu/h)
  - $T_A$ : indoor air temperature (°F)
  - $T_M$ : building mass temperature (°F)
  - $T_o$ : outdoor air temperature (°F)
  - $H_M$ : building mass conductivity to the indoor air (Btu/°F·h)
  - $C_M$ : heat capacity of the building mass (Btu/°F)
  - $C_A$ : heat capacity of the air mass (Btu/°F)
  - $U_A$ : heat conductivity of the building envelop (Btu/°F·h)
- The house thermal dynamics is driven by the following two equations [50]:

$$Q_A - Q_{HVAC} - U_A(T_A - T_o) - H_M(T_A - T_M) - C_A \left( \frac{dT_A}{dt} \right) = 0 \quad (2)$$

$$Q_M - H_M(T_M - T_A) - C_M \left( \frac{dT_M}{dt} \right) = 0 \quad (3)$$

Solving (2) and (3) for  $T_A$  and  $T_M$  gives a closed form solution of function  $f$  as:

$$T_A^{t_{k+1}} = A_1 e^{r_1 \Delta t} + A_2 e^{r_2 \Delta t} + \frac{d}{c} = f(T_{A_n}^{t_k}, T_{M_n}^{t_k}, C_{T_k}^{t_k}, T_o^{t_k}, \Delta t, U_n^{t_k}) \quad (4)$$

$$T_M^{t_{k+1}} = A_1 A_3 e^{r_1 \Delta t} + A_2 A_4 e^{r_2 \Delta t} + g + \frac{d}{c} \quad (5)$$

Variables  $A_1, A_2, A_3, A_4, d, c, r_1, r_2$  are constants related to house thermal parameters, the initial value of air, mass and outdoor temperature, heat gains and the HVAC state. In this study, all of the houses are assumed to be located in the same geographical area, and hence the spatial variance of outdoor temperature has been ignored. The temporal variation of the ambient temperature is included in the simulation, however, it is assumed that the prediction for the next day is available and accurate.

### 3. Proposed solution

The proposed solution for (1) is a binary search algorithm, which uses a novel greedy algorithm to optimally control HVACs. This is discussed below:

#### 3.1. Determination of optimal $D_L$

This algorithm searches for the minimum value of  $D_L$  using binary search. The range of possible solution for  $D_L$  is given by (6).

$$0 \leq D_L \leq \sum_{n=1}^N P_{HVAC_n} \quad (6)$$

The binary search starts with the first trial equal to the middle of the range in (6).

$$D_{L1} = \frac{1}{2} \sum_{n=1}^N P_{HVAC_n} \quad (7)$$

For each trial determines if the aggregated power can be kept below  $D_L$  using a function  $f_o(D_{Lk})$  discussed in Section 3.3. If  $f_o(D_{Lk})$  returns true, the next trial value of  $D_L$  will be chosen to be even lower; and if it returns false, the next trial will be higher. The process is repeated until  $D_L$  varies between iteration by less than 0.1% of the maximum value. The complete algorithm to find optimal  $D_L$  is shown below:

**Algorithm 1.** Finding optimal  $D_L$ .

- 1:  $right = \sum_{k=1}^n P_{hvac_n}$
- 2:  $left = 0$

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3:  $D_{L_0} = \text{right}$ 
4:  $k = 1$ 
5:  $\text{tol} = 0.001 * \text{right}$ 
6: while True:
7:    $\text{mid} = (\text{right} + \text{left})/2$ 
8:   if  $\text{abs}(D_{L_{k-1}} - \text{mid}) > \text{tol}$ :
9:      $D_{L_k} = \text{mid}$ 
10:   else:
11:     break
12:   if  $f_O(D_{L_k})$ :
13:      $\text{right} = D_{L_k}$ 
14:   else:
15:      $\text{left} = D_{L_k}$ 
16:    $k = k + 1$ 
17: end while
18: return  $D_{L_k}$ 

```

Since the time complexity of a binary search algorithm is  $O(\log(N))$ , up to 0.1% of the  $\sum_{k=1}^n Phvac_n$  can be reached within at most 10 iterations ( $\log_2(1000) = 10$ ), which is very efficient.

### 3.2. Optimal HVAC control for a given $D_L$

For function  $f_O(D_{L_k})$  to determine if the aggregated power can be kept below  $D_{L_k}$ , an optimal HVAC control algorithm is described here. This algorithm selects a set of HVACs during each time step, so that the comfort constrained is maintained and the aggregated power remains below the demand limit  $D_{L_k}$ . Fig. 3 Alternate view of the problem.

The vertical position of the (red and blue) balls represent the indoor air temperature for each house, and the black bars on the top and bottom correspond to the upper and lower temperature bounds of respective houses. Different balls have different weights that correspond to the rated power of corresponding HVACs. Considering the cooling mode of operation, the temperature naturally rises because of the outdoor temperature and heat gains, which is analogous to the balls rising. The rise rate is different for different balls as heat gain and insulation are different for different houses. Now, during each time step, there is a choice of bringing down  $M$  out of these  $N$  temperatures (balls) at their respective cooling-down rate, which is different for different houses. The problem then is to repeatedly select  $M$  out of these  $N$  balls to bring down at each time step (and let the rests rise up) so that none of these balls hit the boundary for the maximum possible time.

Now, the greedy algorithm proposed is to keep selecting HVAC with the shortest *time-to-boundary* until their aggregated power reaches the demand limit  $D_L$ . The ‘*time-to-boundary*’ for the HVAC of house  $n$  at a given time  $t_k$  is defined as the time it takes for the temperature to hit the upper limit if no HVAC operation happens. This is equivalent to the time it takes for a ball to hit its upper wall in Fig. 3. The *time-to-boundary*,  $B_n^{t_k}$ , can be calculated using (4) by solving for  $\Delta t$  that would make the temperature hit the upper boundary, as shown in (7):

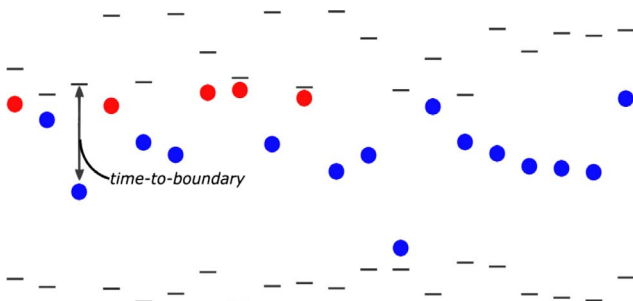


Fig. 3. Alternate view of the problem.

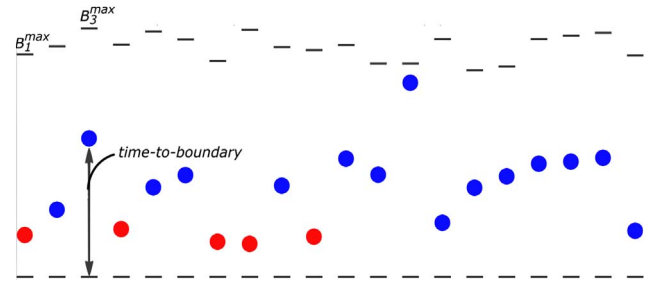


Fig. 4. Transform the problem to represent the time-to-boundary as the height of each ball.

$$\theta_{upper_n} = f(T_{A_n}^{t_k}, T_{M_n}^{t_k}, C^{t_k}, T_o^{t_k}, B_n^{t_k}, 0) \quad \forall n \quad (7)$$

where

$B_n^{t_k}$ : time to boundary for HVAC of house  $n$  at time  $t_k$   
 0: number zero (because the HVAC state is turned off)

Now, instead of looking at individual HVACs in terms of their temperature, upper boundary and lower boundary, they can be viewed in terms of their *time-to-boundary*. This lets us transform Fig. 3 into Fig. 4.

The height of the balls in Fig. 4 now represent their *time-to-boundary*, and the lower solid lines represent the lower limit for *time-to-boundary*, which is 0 for all balls. In this transformed view of problem, all balls fall down at the same rate, except the red balls that are selected to rise up during the current control intervals. By how much the red balls rise depends on their individual house thermal parameters and HVAC cooling capacities. Also, the upper boundary for each ball can be different because the maximum allowable *time-to-boundary* for each ball is different and is denoted by maximum-*time-to-boundary*,  $B_n^{max}$ , which can be determined by solving (7) with the  $T_{A_n}^{t_k}$  and  $T_{M_n}^{t_k}$  replaced by the lower temperature boundary. Also, the weight of each ball can be different owing to different HVAC rated power.

The proposed greedy algorithm then corresponds to selecting the bottom-most balls in Fig. 4 during each control interval until their total weight reaches  $D_L$ . The balls that would hit the upper boundary if selected to rise are skipped in the process. The state of the corresponding HVACs remains constant for each control period, and is updated at the beginning of the next control period. This algorithm is analogous to a showman juggling several tennis balls where, at any given time, only two balls are being thrown up and the rest are falling down, but by cleverly switching between the balls, he manages to keep the height of all balls above the ground. As such, this greedy algorithm for HVAC control is named Juggling Algorithm (JA). The algorithm is as follows:

**Algorithm 2.** Juggling Algorithm (JA) for HVAC control.

```

1: Get  $D_L$ 
2: for each time step  $k$ :
3:   for each HVAC  $n$ :
4:     Calculate  $B_n^{t_k}$  by solving (7)
5:   end for
6:    $\text{sorted\_hvac} \leftarrow \text{sort based on } B_n^{t_k}$ 
7:    $\text{sum} = 0$ 
8:    $\text{full} = \text{false}$ 
9:   for HVAC $_n$  in  $\text{sorted\_hvac}$ :
10:    if  $B_n^{t_k} + D_n \leq B_n^{max}$  and not full:
11:      if  $P_{HVAC_n} + \text{sum} \leq D_L$ :
12:         $U_n^{t_k} = 1$ 
13:         $\text{sum} = \text{sum} + P_{HVAC_n}$ 
14:      else:
15:         $U_n^{t_k} = 0$ 
16:         $\text{full} = \text{true}$ 
17:      else:

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18:    $U_n^{lk} = 0$ 
19: end for
20: end for

```

The  $D_n$  in the above algorithm is the time by which the *time-to-boundary* of HVAC<sub>n</sub> is delayed (increased) when it is controlled in a control period. It depends on house properties and HVAC capacity. Its numerical value can be determined by calculating the difference between two *time-to-boundary* values obtained using (7): one—chosen at the current temperature, and the other—made equal to the temperature attained when the HVAC cools the building for one control period. For the proof of optimality of the Algorithm 2 in Appendix A, it is assumed that  $D_n$  remains constant for any choice of the starting temperature. This is a valid assumption as  $D_n$  has been numerically verified to vary less than 5% when the temperature falls in a narrow region, and is regarded to be insignificant enough to be treated as constant for the proof.

This method of controlling HVACs by holding their state during each control period has also been called deadband free control and is shown effective in eliminating demand drift [51], which is a phenomenon where the aggregated power drifts away from the dispatched power due to natural dispersion of thermostat states [52]. The inspiration for the JA comes from the classical earliest-deadline-first job scheduling algorithm where the *time-to-boundary* is equivalent to deadline for the job of controlling the HVAC. More details on the JA and its proof of optimality are available in Appendix A.

For this study, shortest acceptable HVAC cycling time is assumed to be 5 min and the same is chosen to be the length of the control period. However, the methods developed in this paper would be applicable to any length of the control period. Note that while HVAC should not be cycled arbitrarily fast, the shortest acceptable interval is chosen as it provides the best chance of satisfying the constraint for the longest time. This is because, smaller time steps enable the algorithm to switch between different HVACs more frequently thereby allowing indoor temperatures to remain within the bounds for longer time.

### 3.3. Determination of function $f_o$

Algorithm 3 shown below serves to complete the function  $f_o(D_{L_k})$  that determines if the HVAC control dictated by Algorithm 2 can satisfy the comfort requirement of all customers (i.e., maintaining all house temperature within bounds) while keeping the aggregated HVAC power below  $D_{L_k}$ .

**Algorithm 3.** Algorithm for  $f_o(D_{L_k})$ .

```

1: for each time step k:
2:   for each HVAC n:
3:     Calculate  $T_{An}^{lk}$  using equation(4)
4:     if not  $\theta_{lower_n} \leq T_{An}^{lk} \leq \theta_{upper_n}$ :
5:       return false
6:     else:
7:       Assign  $U_n^{lk}$  using Algorithm 2
9:   end for
10: end for
11: return true

```

At the beginning of each control period during a DR event, the temperature of each HVAC is calculated using (4). If any of the temperature is found to violate the comfort constraint, then it is deemed unfeasible to meet the requirement of keeping the power below  $D_{L_k}$  and the function returns false. If all temperature is found to be within the comfort constraints, then the set of HVAC that needs to be turned on are determined using Algorithm 2. Their states are updated to ON, and the

algorithm moves to the next control period. The process repeats, and if, by the end of the DR event, none of the temperature violates the comfort constrain, it is deemed feasible to keep the power below  $D_{L_k}$  and the function returns true.

## 4. Simulation study

A simulation study was conducted using SimPy [53] – a python based discrete event simulation library where system states are only updated at discrete times [54]. The thermal model of a residential house was developed following the simplified version of the house\_e model in GridLAB-D [50,55]. There are two sources of internal heat gain in a residential building: from appliance use and from human occupancy. In this study, internal heat gain from human occupancy and appliance use are modelled using a single variable and are calculated from the house floor area as per [50,55].

The study was conducted on the aggregation of 200 residential houses assumed to be located in Chicago, Illinois. Typical metrological year outdoor temperature and solar insolation data for the same location were used. Thermal parameters for houses were randomized as follows:

- Floor area was randomized using a normal distribution with the mean area of 2200 square feet and the standard deviation of 400 square feet.
- Aspect ratio of each house was varied uniformly from 1.2 to 1.8.
- R values for windows were varied normally with mean of 1/0.6 and standard deviation of 0.2. R values of doors were varied between 4 and 6.
- Air change was varied from 40% to 80% per hour.
- AC rated capacities were calculated based on floor area [55,50], and rounded to nearest 6000 BTU/h.

The simulation study is conducted on a computer with core-i7 3280 3.6Ghz CPU and 16 GB RAM. For the case with 200 houses, the Algorithm 1 took 2.29 s to find the optimum  $D_L$ , while each step of Algorithm 2, which is used to find the state of HVACs during each control interval in real-time, took only 0.009 s per step. In order to confirm the growth order (the time complexity) of these algorithm, simulation was also conducted for 50, 100, 500, 1000, 2000 and 4000 houses and the solution time measured. The result is demonstrated in Fig. 5. It can be seen that the solution for real-time scheduling can be obtained in sub-second time even for large number of houses. This demonstrates the feasibility of implementing this algorithm in the real

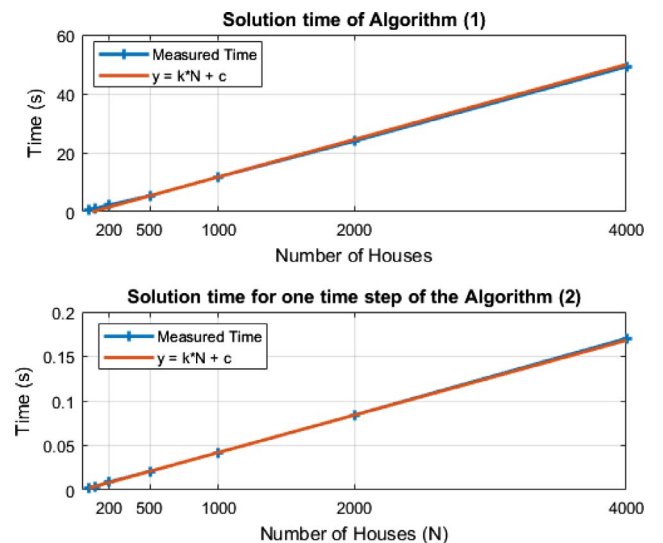


Fig. 5. Solution Time growth rate with problem size.

world. The optimal demand limit can also be found out moderately quickly (under a minute), and since the DR requirement is generally communicated several hours in advance by the utility, it makes the algorithm feasible to be used to find the optimal load reduction as well. The measured linear growth order is consistent with the theoretical growth order that can be inferred from the structure of the algorithms.

The study compares the proposed JA based load control approach with the set-point change based method which is a standard load control method in a transactive control based approach. To make the comparison fair, the upper and lower comfort boundaries during the event were kept the same for both approaches, and were chosen to be 82°F and 72°F respectively. This comfort boundary matches the 22–28 °C boundary used in [26] and is based on the ASHARE standard [56]. In order to help the customers, make rational choice about their comfort limits, the aggregator can provide some charts that gives guidelines based on comfort vs savings. To ensure the temperature did not exceed the upper comfort boundary of 82°F during the event, for set-point change based method, the thermostat set-point was set to 81°F with 1°F deadband. During the non-DR period, the set-point was kept at 77°F. The simulation was conducted for a day in August and the DR event was assumed to start from 14:00 to 18:00.

#### 4.1. Base case: no control

In the case without any control, HVAC set-points of all houses were held constant at 77°F throughout the day. The aggregated HVAC power consumption of all 200 houses, together with outdoor and indoor temperature profiles, are shown in Fig. 6. As shown, the aggregated HVAC power starts close to zero in early morning and gradually rises in the afternoon owing to increasing outdoor temperature; and shaves off towards the evening as the temperature falls. It is worth noting that the peak power is 355.4 kW at around 14:47. Indoor temperatures of each house remains within 1°F deadband around the 77°F set-point, except for the early morning period when the temperature is low due to cooler outdoor temperature, and no HVACs run during this period as is evident by zero power consumption before 5 a.m.

#### 4.2. Case I: Juggling algorithm (JA)

By applying the proposed JA discussed in Section 3, the optimal demand limit from 14:00 to 18:00 was found at 118.87 kW. The result of controlling the thermostats according to the JA is shown in the Fig. 7. As shown in the close-up, the aggregated power has been cleanly limited to the calculated optimal value. During the DR event period, the minimum aggregate HVAC power is 115.14 kW which is 3.1% less (or 3.73 kW less) than the limit. This is considered an acceptable tolerance

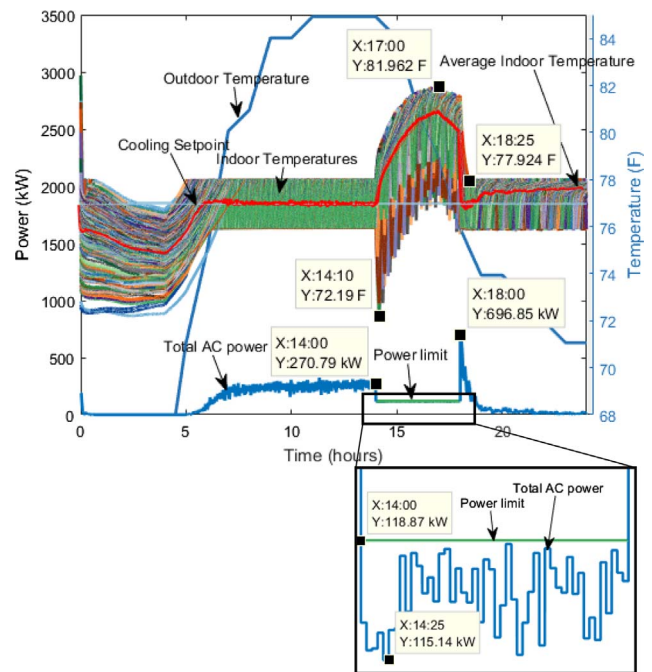


Fig. 7. Aggregated HVAC power, outdoor and indoor temperatures with the JA (Case I).

and is equivalent to the power of one HVAC unit.

Indoor temperatures of individual houses are identical to the base case up until the start of the DR event, at which point indoor temperature start rising. Notice that temperatures of some houses actually fall at the beginning because the HVACs that are selected to run during the first control period operate for the whole 5-min control intervals (as long as the house temperatures do not go below their lower comfort boundaries). Towards the end of the DR event, almost all houses' indoor temperatures approach their upper boundaries (but none exceeds it), signifying the optimal use of resources.

There is a prominent peak in the aggregated HVAC power after the DR event ends, and this is a consequence of resuming the set-point based control at 77°F, which is much lower than houses' indoor temperature at the end of the DR event. This problem is tackled in Case III discussed in sub-Section D.

#### 4.3. Case II: Set-point change based control

In the case of set-point change based control, set-points of all thermostats were raised during the DR event to achieve the desired power reduction. The result is shown in Fig. 8.

While there is an immediate power reduction (to 0 kW) at the start of the DR event, the aggregated power creeps up and reaches the peak of 223.45 kW, which is much higher than the case of the JA at 118.87 kW. Also the average indoor temperature are higher than that in Case I and they reach the upper limit of 82°F much sooner too. The reason for better performance of Case I than this case is because, in this case, during the initial stage of the DR event, the aggregated power remains zero (because the temperatures of all thermostats are much lower than their set-point of 81°F), but during the later stage, much higher power is required to keep the temperature within boundary. In case I, however, the power consumption of 118.87 kW is always maintained, which is used for pre-cooling some of the houses at the beginning stage of the DR which becomes valuable at the later stage. It should be noted that this pre-cooling phenomenon is an inherent feature of our JA and no special programming is required for this.

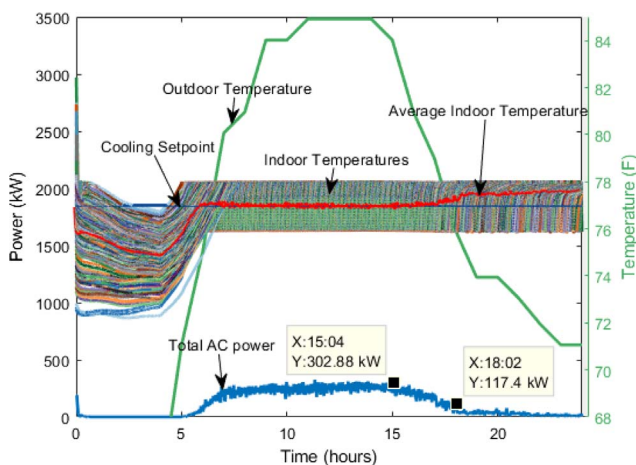


Fig. 6. Aggregated HVAC power, outdoor and indoor temperatures in the base case without control (Base Case).

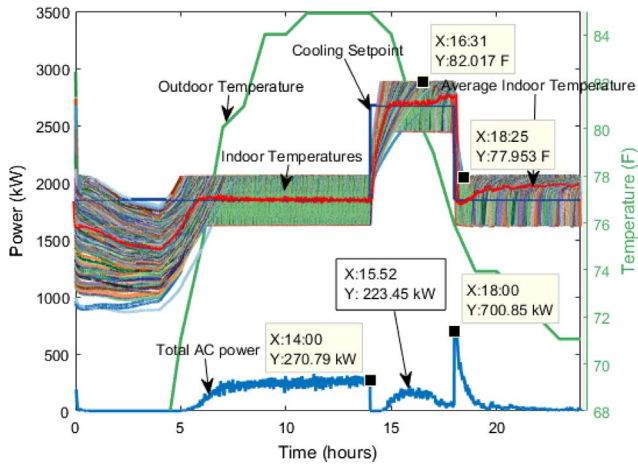


Fig. 8. Aggregated HVAC power, outdoor and indoor temperatures with the set-point change based control (Case II).

#### 4.4. Case III: JA with demand restrike mitigation methods

Two approaches are explored for limiting the demand restrike seen in Case I. They are explained below:

##### 4.4.1. Using demand restrike limit

A demand limit can be imposed on the aggregated power after the DR event using the same JA. This DR limit can be for a duration of  $D_T$  such that the demand restrike is prevented from occurring. The value of  $D_T$  can be conservatively set equal to at least half of the event duration or its optimal value can be obtained by performing an iterative binary search similar to Algorithm 1 that is used to find optimal  $D_L$ . As a case study, the result of limiting the aggregated power to the pre-DR event value of 270.79 kW for up to 35 min after the event is shown in Fig. 9.

Fig. 9 illustrates that the restrike has been successfully suppressed at the pre-DR event level of 270.79 kW. The recovery of indoor temperatures has been delayed, however. While indoor temperatures of all houses return to the 74°F set-point  $\pm 1^\circ\text{F}$  deadband in 25 min in Case I, it now 42 min in Case III. This is the price paid to suppress the demand restrike.

##### 4.4.2. Using conservative demand limit (CDL)

One might think that the demand restrike is associated with the low value of demand limit, and as such it is tempting to use a more liberal demand limit in hope of decreasing the restrike. One such case where the DR limit is made equal to the 1.25 times the optimal value used in

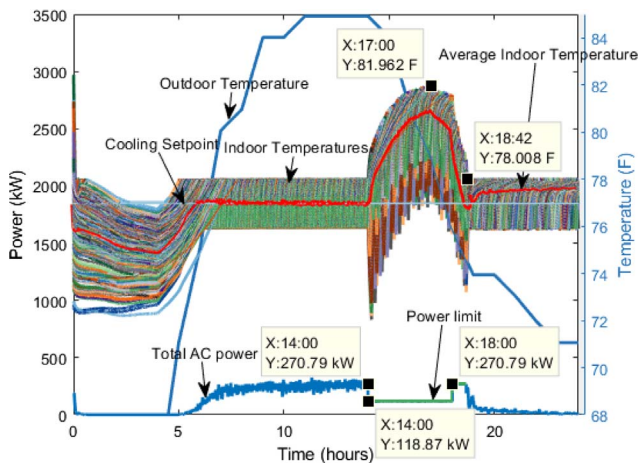


Fig. 9. Aggregated HVAC power, outdoor and indoor temperatures with JA and a restrike limit (Case III.1).

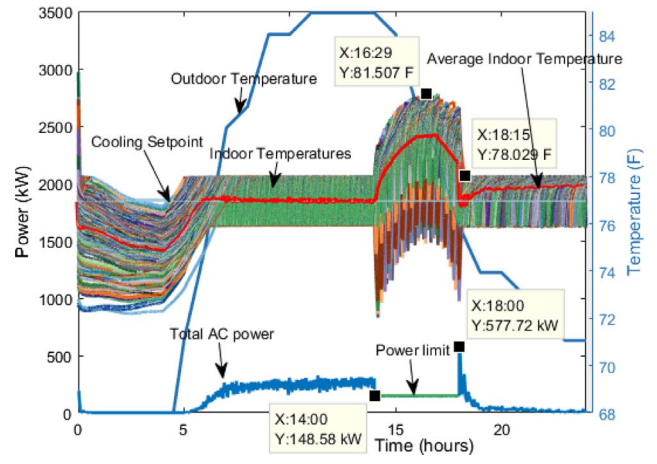


Fig. 10. Aggregated HVAC power, outdoor and indoor temperatures with JA with conservative demand limit (Case III.2).

Case I is shown in Fig. 10.

It can be observed that the demand restrike only decreases from 696.85 kW to 577.72 kW even when the demand limit was increased. The indoor temperatures also never reach near the upper boundary of 82°F (hinting non-optimal utilization), and they quickly return back to the normal values. This demonstrates that increasing the DR-limit imposed during the DR-event does not significantly help with reducing the demand restrike, as the restrike occurs because of loss of state diversity [52].

#### 4.5. Case IV: Effect of randomized constraints

In all case studies so far the desired set-points of all houses were assumed at 77°F and the upper temperature boundary was assumed at 82°F. To understand the impact of randomizing these parameters on the efficacy of the proposed approach, a simulation was conducted that randomized these parameters. The result of one such simulation which is similar to Case III.1 except for the randomized constraints is shown in Fig. 11. Interestingly, the results remain almost the same.

The aggregated power and the average indoor temperature follows almost the same profiles as Case III.1. Even the individual temperatures, although much different than those in Case III.1 at first glance, follow similar profiles in terms of their own set-points and boundaries. This demonstrates that the study so far is directly applicable for the real-world case where houses have different preferred set-points and comfort preference.

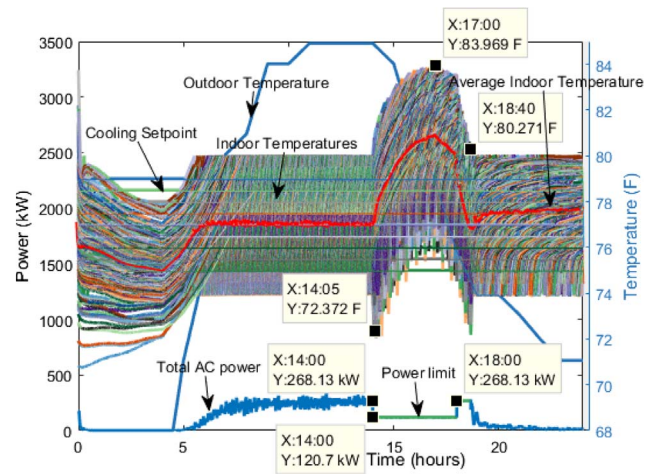


Fig. 11. Aggregated HVAC power, outdoor and indoor temperatures with JA, demonstrating randomized constraints (Case IV).

**Table 1**  
Summary of simulation results.

| Control method | Peak power during DR (kW) | Peak power after DR (kW) | Time to normal temp | Comfort violation degree Hours (°F-h) | Energy Consumption (MWh) | Average AC cycles per day |
|----------------|---------------------------|--------------------------|---------------------|---------------------------------------|--------------------------|---------------------------|
| No Control     | 302.88                    | 117.4                    | –                   | 2.10                                  | 2.85                     | 66.63                     |
| Case I         | 118.87                    | 696.85                   | 00:25               | 12.78                                 | 2.73                     | 58.75                     |
| Case II        | 223.45                    | 700.85                   | 00:25               | 16.21                                 | 2.70                     | 61.64                     |
| Case III.1     | 118.87                    | 270.79                   | 00:42               | 13.33                                 | 2.72                     | 59.17                     |
| Case III.2     | 148.58                    | 577.72                   | 00:15               | 9.57                                  | 2.76                     | 59.77                     |
| Case IV        | 120.7                     | 268.13                   | 00:40               | 13.24                                 | 2.74                     | 59.80                     |

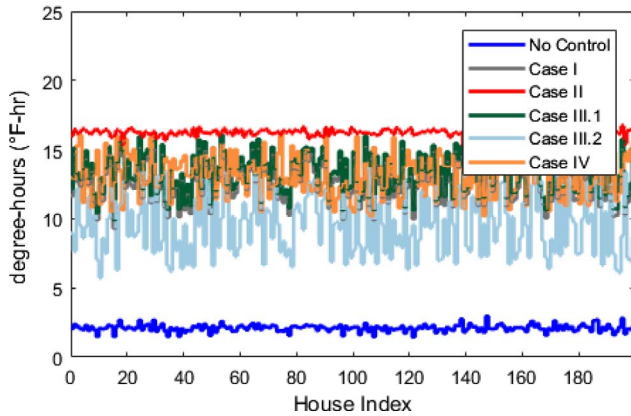


Fig. 12. Distribution of comfort violation index among the houses.

#### 4.6. Summary and other observations

Results of all case studies are summarized in Table 1.

From the table, it can be seen that the peak power during DR is reduced in all cases – but the proposed JA yields the optimal control, i.e., limiting the peak power to around 118 kW (Cases I, III.1 and IV).

As the proposed algorithm could cause demand restrike, mitigating this problem by extending the JA after the DR event (Case III.1) proves to reduce the restrike by more than half (i.e., from 696 kW to around 270 kW).

After the DR event, it takes some time for the temperatures to get back to their regular values for all cases. It is 25 min with the proposed JA (Case I), and 42 min when mitigating the impact of demand restrike (Case III.1). This delayed temperature recovery is the price paid for the restrike mitigation. The discomfort associated with the demand response can be quantified using comfort violation index, which measures the degree-hour time integral of the absolute difference between the desired temperature (which is 77°F) and the actual temperature [43]. The fifth column gives the average comfort violation index for the time interval from 14:00 to 19:00 as that is the time interval influenced by the DR. As can be seen, the setpoint change based control in Case II has the worst performance. The distribution of the comfort violation among different houses is shown in Fig. 12:

Fig. 12 shows that proposed algorithm (Case III.1) keeps the comfort violation below the setpoint control (Case II) based method for all the cases. But unlike the setpoint control based method, there is quite a variation among houses. This is because, unlike in [43] where even distribution of comfort violation was one of the objective, the objective here is to maximize the load reduction while only meeting the comfort

requirement. As such, the algorithm might exploit some houses more than the others to maximize the load reduction. In order to make this fair to the customers, the aggregator can make the rewards proportional to the comfort violation endured by the house.

When looking at the energy consumption (kWh) in Table 1, all DR cases provide lower energy consumption than the base case. This is because the comfort requirement is relaxed during the DR event which allows HVAC to run less frequently during the event, thus saving energy.

In terms of average AC cycles, it is interesting to see that the average number of AC cycles actually decreases in all cases compared to the base case. It reduces to an average of 59.17 cycles with Case III.1 compared to average of 66.63 cycles in the base case. This decrease is possibly because during DR event a large fraction of the HVACs remains off and only a small fraction is selected to run during each control interval. This shows that the proposed approach will not result in reduced operating life of the AC due to increased AC cycling.

Although the problem was formulated to make the aggregated power below the maximum limit,  $D_L$ , it has been observed that the aggregated power is always maintained very close to this limit. As such the work can be extended for applications where the aggregated power needs to track a reference value.

#### 5. Conclusion

The novel algorithm for an advanced direct load control mechanism for HVAC presented in this paper was found to successfully reduce the peak power by up to 60% while keeping indoor temperatures within preset limits. This is much better than typical set-point based control that reduced the power demand by 26% under the same temperature limits. This is an improvement of 130% in demand reduction potential. The proposed algorithm was also successful in efficiently limiting the DR restrike, a feature not typically available in other approaches. Since, in terms of hardware, the proposed algorithm only requires the presence of IoT-based smart thermostats in buildings, demand aggregators can readily use it without additional infrastructures. This can be a great business opportunity for aggregators to sell demand response capability as a service to the grid.

#### Acknowledgment

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#### Appendix A

Consider the transformed view of the problem in Fig. 4. Without loss of generality, let us assume that the control period ( $T$ ) from  $t_{start}$  to  $t_{end}$  be divided into  $Z$  intervals of length  $L$ , which is 5 min, so that  $Z = T/L$ . During each control period, JA selects a subset of HVACs to run according to Algorithm 2.

At the beginning of any control period  $k$ , the *time-to-boundary* for HVAC  $n$  that has previously been ran  $x$  times can be obtained as:

$$B_n^{tk} = B_n^h + D_n * x - L * (k-1-x) = g_n(k, x) \quad \forall n \quad (A1)$$

where

$t_k$  is the control period of concern

$B_n^{tk}$  is the *time-to-boundary* for the HVAC  $n$ , at the beginning of control period  $t_k$

$B_n^h$  is the *time-to-boundary* for the HVAC  $n$ , at the beginning of DR event

$D_n$  is the time by which the *time-to-boundary* of HVAC $_n$  is delayed when it is controlled in a control period (defined in Algorithm 2)

$x$  is the number of times HVAC  $n$  has already run by control period  $t_k$

$L$  is the control period length (5-min)

The objective is to ensure that, at all control periods, indoor temperatures of all houses are within the bounds, which is equivalent to ensuring that

$$0 \leq B_n^{tk} \leq B_n^{max} \quad \text{for } 1 \leq k \leq Z + 1$$

This enables us to reformulate this problem as a job scheduling problem [57]. Let us define a job  $J_n^i$  as the task of running HVAC  $n$  for the  $i$ -th time. The job  $J_n^i$  can be delayed until the time to boundary for HVAC  $n$  would become negative. Hence, the deadline ( $d_{J_n^i}$ ) for job  $J_n^i$  can be obtained as

$$d_{J_n^i} = \max_k g_n(k, i-1) > 0$$

$$d_{J_n^i} = \left\lfloor \frac{B_n^h + (D_n + L) * (i-1) + L}{L} \right\rfloor$$

Similarly, the job  $J_n^i$  can be scheduled as soon as its time to boundary does not exceed  $B_n^{max}$  when controlled. That is, the release time ( $r_{J_n^i}$ ) can be obtained as:

$$r_{J_n^i} = \min_k g_n(k, i-1) + D_n \leq B_n^{max}$$

$$r_{J_n^i} = \left\lceil \frac{B_n^h + (D_n + L) * i - B_n^{max}}{L} \right\rceil$$

Hence, the deadlines and release times for  $J_n^i$  is fixed and independent of how other jobs are scheduled. The only constraint is the precedence constraints where Job  $J_n^i$  must occur strictly before job  $J_n^{i+1}$ . It should be noted that a feasible schedule is an optimal schedule in our case, because if the optimal schedule were to finish scheduling the jobs earlier than the feasible schedule, the number of parallel jobs would be decreased in the next iteration of the Algorithm 1 and so-on until only the optimal schedule remains feasible. In rest of the Appendix A, feasible and optimal is used interchangeably when talking about the schedule.

Now, let the set of jobs selected to run during the interval  $i$  by the optimal scheduling algorithm (O) be  $O_i$ , and that by the proposed JA (A) be  $A_i$ .

Let JA agree with the optimal algorithm for up to some control period  $k$  such that  $0 \leq k < n$ . That is,

$$O_i = A_i \quad \text{for } i < k$$

$$O_i \neq A_i \quad \text{for } i = k$$

Let  $F$  jobs be scheduled during the interval  $k$  by the optimal algorithm and be given by:

$$O_k = \{O_{k,1}, O_{k,2}, \dots, O_{k,F}\}$$

And let  $G$  jobs be scheduled during the interval  $k$  by JA and be given by:

$$A_k = \{A_{k,1}, A_{k,2}, \dots, A_{k,G}\}$$

These jobs are sorted in the ascending order of their *deadlines*. Note that  $F \neq G$  since the number of jobs scheduled during each control period by the algorithms can be different because of different weights. Also, it is straightforward to show that JA as defined in Section 3 is equivalent to selecting jobs with earliest deadlines that have been released such that the sum of their weight is just less than the demand limit. It is assumed that for all  $k$ , due to the weight constraint, there is no more room for any other job in  $A_k$ .

The algorithm A is now proved to be optimal using an exchange argument [58], which is a proof technique where a test algorithm is proved optimal by gradually modifying the optimal algorithm and finally making it the same as the test algorithm, and never losing the optimality in the process. Below, it is shown that the optimal algorithm can be modified into the JA without losing its optimality.

$$\text{Let } A_k - O_k = \{A_{k,j}, A_{k,j+1}, \dots, A_{k,F}\} = \{J_{p_j}^{x_j}, J_{p_{j+1}}^{x_{j+1}}, \dots, J_{p_F}^{x_F}\}$$

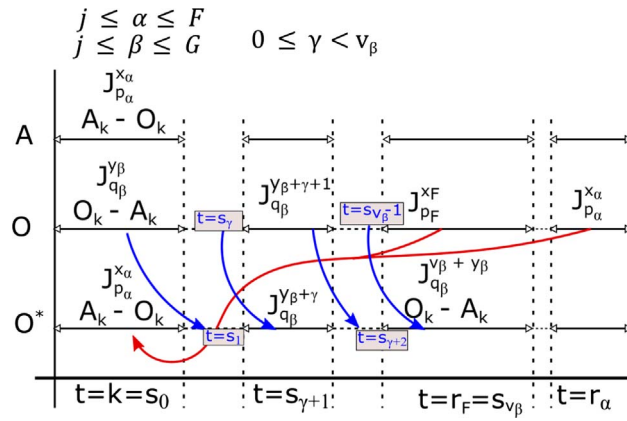
$$\text{And } O_k - A_k = \{O_{k,j}, O_{k,j+1}, \dots, O_{k,G}\} = \{J_{q_j}^{y_j}, J_{q_{j+1}}^{y_{j+1}}, \dots, J_{q_G}^{y_G}\}$$

... sorted in the ascending order of their *deadlines*.

$A_k - O_k$  cannot be empty because if  $O_k$  contains all jobs of  $A_k$  then it cannot have any space for more jobs and it would be exactly equal to  $A_k$  negating our presumption of that  $O_i \neq A_i$  for  $i = k$ .  $O_k - A_k$  can be empty, though.

Let us assume that  $J_{p_\alpha}^{x_\alpha}$  (for all  $j \leq \alpha \leq F$ ) occurs at some later time at  $t = r_\alpha$  at  $O$ . Or, it does not occur at all in  $O$ . For example,  $J_{p_F}^{x_F}$  if present, occurs at  $t = r_F$ . If it does not exist, it is assumed to exist after the control period, so that  $r_F = Z + 1$ .

Assume that for HVAC  $q_\beta$  (for all  $j \leq \beta \leq G$ ), there are  $v_\beta$  number of jobs that occur between time  $k \leq t < r_F$  in  $O$ . These jobs occur at times  $t = s_\gamma$  (for all  $0 \leq \gamma < v_\beta$ ) and are denoted by  $J_{q_\beta}^{y_\beta + \gamma}$ . This includes the job  $J_{q_\beta}^{y_\beta}$  at  $t = k$ , and this means  $s_0 = k$  (because  $J_{q_\beta}^{y_\beta + 0}$  is assumed to run at  $t = s_0$ ). It is possible that  $v_\beta = 1$ , which means only  $J_{q_\beta}^{y_\beta}$  job occurs for HVAC  $q_\beta$  in  $O$  during that time.

Fig. A1. Illustration of the exchange step at time step  $t = k$ .

Now, create a new schedule  $O^*$  by modifying the schedule  $O$  as illustrated in the Fig. A1. The jobs in  $A_k - O_k$  are moved from various places in  $O$  (if they exist in  $O$ ) to  $t = k$  in  $O^*$ . If any of the jobs in  $A_k - O_k$  does not exist in  $O$ , they are simply created at  $t = k$  at  $O^*$  (instead of moving).

All the jobs  $J_{q_{\beta}}^{\beta+\gamma}$  are moved from  $t = s_{\gamma}$  in  $O$  to  $t = s_{\gamma+1}$  in  $O^*$  and  $s_{v_{\beta}}$  is defined to be equal to  $r_F$ .

It is next shown that these changes still ensure that all jobs are run after release times and before deadlines in  $O^*$  (if it does in  $O$ , and it should since it is assumed to be an optimal (feasible) algorithm).

The jobs  $A_k - O_k$  are run before deadline in  $O^*$  at  $t = k$ , since they were run later than  $t = k$  (or not run at all) in  $O$ . Also, they will run after release in  $O^*$  since, they run in  $A$  at  $t = k$ , and  $A$  only runs jobs after their release time.

Jobs  $J_{q_{\beta}}^{\beta}$  have deadline later than any jobs in  $A_k$  because  $J_{q_{\beta}}^{\beta}$  are not present in  $A_k$  and  $A_k$  has jobs with earliest deadlines that are released by  $t = k$  (and  $J_{q_{\beta}}^{\beta}$  are released by  $t = k$  since they are present in  $O_k$ ). Job  $J_{p_F}^{x_F}$  is in  $A_k$  and it runs at  $t = r_F$  in  $O_k$ , so jobs  $J_{q_{\beta}}^{\beta}$  must have deadline after  $t = r_F$ . Since for any job  $d_{j+1} > d_{j1}$ , all jobs  $J_{q_{\beta}}^{\beta+\gamma}$  have deadline after  $t = r_F$  and are guaranteed to run before deadline in  $O^*$  since they are run at or before  $t = r_F$ .

Also all jobs  $J_{q_{\beta}}^{\beta+\gamma}$  are run later in  $O^*$  than in  $O$ , so they are guaranteed to be run after release.

Since all other jobs in  $O$  are left unaffected in  $O^*$ , they must continue to run before deadline and after release. Hence,  $O^*$  is still feasible schedule.

When this process is completed:

$$O_i^* = A_i \text{ for } i \leq k$$

$$O_i^* \neq A_i \text{ for } i = k + 1$$

That is,  $O^*$  agrees with  $A$  for one more time step than  $O$ , and  $O^*$  still runs all jobs before deadline and after release time. It should be noted that  $O^*$  may not meet the weight constraint at this point, but that is fine since this is only the intermediate step of the exchange process. This procedure can then be repeated for all future time steps, such that  $O_i^* = A_i$  for all  $i$ , which means  $O^* = A$ . Since  $O^*$  always remains feasible, finally when  $O^*$  becomes equal to  $A$ , it should remain feasible and also meet the weight constraint because  $A$  does. It was argued before that a feasible algorithm is optimal in our case. This concludes the proof that  $A$  is optimal.

## Appendix B. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.apenergy.2018.02.085>.

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