Equivalent Ramp Rate Function for Thermal Power Systems

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Abstract—**The equivalent modeling method is extensively used in the area of power system optimization. The thermal power system can be simplified into an equivalent thermal unit in the studies where the performance of individual units is not the focus. Conventionally, the equivalent ramp rate for the equivalent thermal unit is the summation of each unit's ramp rate. However, the equivalent ramp rate relies heavily on the operating points of each thermal unit. In this paper, a concept termed the equivalent ramp rate function (ERRF) for thermal power systems is proposed. Given the principle of dispatch such as cost minimization, the impact of each thermal unit's operating point is accurately considered in ERRF. An algorithm is proposed to numerically build the ERRF. The case study based on the IEEE 30-bus system indicates that the proposed ERRF can reflect *more realistically* the system-wide ramping capability. For example, the ERRF can be utilized as an "offline" evaluation tool to assess the potential wind curtailment due to lack of ramping flexibility. The impact of different cost functions on the shapes of ERRF is also presented.**

Index Terms—**Economic dispatch (ED), equivalent ramp rate function (ERRF), wind curtailment, flexibility.**

NOMENCLATURE

Variables

P_g^t	Scheduled output of generator g in period t
P_s^t	Thermal system output in period t
W^t	Wind power in period t
D^t	System load in period t
$P_{g,\max}$	Maximum capacity of generator g
$P_{g,\min}$	Minimum stable generation (MSG) of generator g
$P_{g,up}$	Ramp-up rate of generator g
$P_{g,dn}$	Ramp-down rate of generator g

Constants

d	Step-length of the thermal system output

Sets

G	Set of generators
T	Set of operating snapshots

Functions

$f_g(\cdot)$	Operating cost function of generator g
$f(\cdot)$	Equivalent cost function for thermal power system
$\mathfrak{R}_{up}(\cdot)$	Equivalent ramp-up rate for thermal power system
$\mathfrak{R}_{dn}(\cdot)$	Equivalent ramp-down rate for thermal power system

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I. INTRODUCTION

THE equivalent modeling method is extensively used in the area of power system optimization. For instance, an equivalent thermal power station has been introduced to a purely thermal economic dispatch problem in [1]. In the hydro-thermal coordination problem, a number of power stations can be substituted with a single power station that behaves equivalent to the entire set [2]. Doing so significantly simplifies the hydrothermal models. In these studies, the performance of individual generating units is not the focus, more emphasis is put on the system-wide performance. In addition, due to the memory limit or computational efficiency concern, sometimes the simplification and equivalent modeling is desirable.

The existing approaches mainly focus on the equivalent modeling of the cost function and generation limits. The generation limits of the equivalent thermal unit is the summation of each unit's generation limit. The equivalent cost function for thermal power systems can be obtained using a convolution method proposed in [3]. The theoretical foundation is that thermal units provide active power and are generally not limited in how they do this, i.e. magnitude and duration [4]. With the increase of wind penetration in power systems, the variability of net loads is likely to increase in future. For instance, Electric Reliability Council of Texas (ERCOT) has experienced one hour ramp-up of 3,039 MW and one hour ramp-down of 2,847 MW in 2009 [5]. In such a power system, wind curtailment might occur due to lack of ramping flexibility. Thermal units' ramp rates are constrained by various factors.

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Therefore, it becomes more important to model ramping flexibility accurately [6].

Much effort has recently been dedicated to power system flexibility. The index of flexibility for both individual thermal units and the whole thermal power system is defined in [7]. Conventionally, the system-wide ramp rate is the summation of the units' ramp rates [8]. However, the ramp rate relies heavily on the operating points of thermal units. The simply summation of the units' ramp rates may produce inaccurate results. Therefore, there is still a need to investigate the modeling technique for system-wide ramp rates.

To this end, this paper proposes a new concept termed the *equivalent ramp rate function* (ERRF) for thermal power systems. In this work, the focus is on thermal generation flexibility. ERRF quantifies the relationship between the system-wide ramp rate and thermal system output. A model and algorithm for numerically formulating the ERRF is presented. Given the principle of dispatch such as cost minimization, the impact of thermal units' operating points on the system-wide ramp rate can be accurately considered. Case studies reveal that the proposed ERRF can better reflect the system-wide ramp rates.

Rest of the paper is organized as follows. Section II presents the conventional dynamic economic dispatch model. Section III introduces the equivalent ramp rate function model (noted as the ERRF model). The case study based on the IEEE 30-bus system is presented in Section IV. Section V has the conclusion.

II. Conventional Dynamic Economic Dispatch Model

The conventional dynamic economic dispatch (DED) model is expressed as follows.

$$\min_{P_g^t} \sum_{t\in T}\sum_{g\in G} f_g\left(P_g^t\right) \tag{1}$$

s.t.

$$\sum_{g\in G} P_g^t + W^t = D^t, t\in T \tag{2}$$

$$-P_{g,dn} \le P_g^t - P_g^{t-1} \le P_{g,up},\ g\in G, t\in T \tag{3}$$

$$P_{g,\min} \le P_g^t \le P_{g,\max},\ g\in G, t\in T \tag{4}$$

The decision variables are each units' scheduled output P_g^t. The optimization objective is to minimize the operating cost as equation (1). The cost function can be either a linear or a quadratic function. The constraints include the power balance equations (2), the ramping constraints (3) of thermal units, the generation limits (4) for thermal units. For simplicity, the transmission loss and network constraints are not considered in this model.

Partial loaded synchronized generating units can provide ramp-up flexibility that is limited by their ramp-up rate and the spare capacity between their scheduled output P_g^t and their maximum capacity $P_{g,\max}$. Likewise, their ramp-down rate and the difference between their scheduled output P_g^t and their minimum stable generation (MSG) $P_{g,\min}$ limit their ability to provide ramp-down flexibility [7]. Mathematically, the effective ramping capability that a thermal unit can provide for system operations is shown as follows:

$$P_g^t - P_g^{t-1} \le \min\left\{P_{g,up}, P_{g,\max} - P_g^{t-1}\right\},\ g\in G, t\in T \tag{5}$$

$$P_g^{t-1} - P_g^t \le \min\left\{P_{g,dn}, P_g^{t-1} - P_{g,\min}\right\},\ g\in G, t\in T \tag{6}$$

In the conventional DED model, since each thermal units are explicitly modeled, the impact of units' operating points on the system-wide ramp rate can be accurately considered. However, sometimes, due to the memory limit or computational efficiency concern, the simplification and equivalent modeling is desirable. In that case, the equivalent ramp rate of thermal power systems needs to be investigated.

III. Equivalent Ramp Rate For Thermal Power Systems

The objective function is to minimize the operating cost of thermal power systems, which is shown as follows.

$$\min_{P_s^t} \sum_{t\in T} f\left(P_s^t\right) \tag{7}$$

where, $f\left(\cdot\right)$ is the equivalent thermal cost function of all the thermal generators. The details of constructing the cost function can refer to [3]. The decision variable is the thermal system output P_s^t.

A. Conventional Formulation

As aforementioned, conventional formulation of the system-wide ramp rate is the simply summation of the units' ramp rates.

$$-\sum_{g\in G} P_{g,dn}^t \le P_s^t - P_s^{t-1} \le \sum_{g\in G} P_{g,up}^t,\ g\in G, t\in T \tag{8}$$

In this formulation, it is implicitly assumed that each unit can provide full ramp-up capability until the thermal system output reaches above $(\sum_{g\in G} P_{g,\max} - \sum_{g\in G} P_{g,up})$. Similarly, it is implicitly assumed that each unit can provide full ramping down capability only when the thermal system output decreases below $(\sum_{g\in G} P_{g,\min} + \sum_{g\in G} P_{g,dn})$. Otherwise, the system-wide ramp-up/ramp-down rates will always be $\sum_{g\in G} P_{g,up}$ and $\sum_{g\in G} P_{g,dn}$, respectively. However, given the principle of dispatch such as cost minimization, units are scheduled according to the merit order. Some units may reach their full capacity earlier than others. Therefore, the system-wide ramp-up rate will be less than $\sum_{g\in G} P_{g,up}$ before the thermal system output reaches above $(\sum_{g\in G} P_{g,\max} - \sum_{g\in G} P_{g,up})$. In other words, the conventional approach may overestimate the ramping flexibility of the system, leading to *optimistic* judgment.

B. Proposed Formulation

The *equivalent ramp rate function* (ERRF) for thermal power systems can be expressed as follows.

$$-\Re_{dn}\left(P_s^{t-1}\right) \le P_s^t - P_s^{t-1} \le \Re_{up}\left(P_s^{t-1}\right),\ g \in G, t \in T \quad (9)$$

Instead of constant ramp rates in (8), the system-wide ramp rate is a function of the thermal system output P_s^{t-1}, which is the aggregation of each unit's operating point.

$$\Re_{up}\left(P_s^{t-1}\right) = \sum_{g \in G}\left(\min\left\{P_{g,up}, P_{g,\max} - P_g^{t-1}\right\}\right), t \in T \quad (10)$$

$$\Re_{dn}\left(P_s^{t-1}\right) = \sum_{g \in G}\left(\min\left\{P_{g,dn}, P_g^{t-1} - P_{g,\min}\right\}\right), t \in T \quad (11)$$

Theoretically, the system-wide ramp rates can be expressed as (10) and (11), respectively. However, in the equivalent model for thermal power systems, individual thermal unit is not explicitly modeled. The decision variable is merely the thermal system output P_s^t. It is convenient and required to express the system-wide ramp rate using the variable P_s^t. The subsequent task is to quantify the relationship between the system-wide ramp rates $\Re_{up}\left(P_s^{t-1}\right)$ and $\Re_{dn}\left(P_s^{t-1}\right)$ and the thermal system output P_s^{t-1}.

With the increase of thermal system output, some units may reach their full capacity earlier than others. These binding generating units cannot provide ramp-up capability for system operations. Therefore, the system-wide ramp-up rate would decrease with the increase of the thermal system output. Likewise, with the decrease of thermal system output, some units may reach their minimum stable generation earlier than others. These binding generating units cannot provide ramp-down capability for system operations. Therefore, the system-wide ramp-down rate would decrease with the decrease of the thermal system output. This will be verified in Section IV.

C. Algorithm

Since it is very difficult to express the ERRF (10)-(11) in an explicit manner, the following algorithm is devised to numerically build the ERRF.

For simplicity, the economic dispatch model at period t is taken as an example.

The objective function is as follows.

$$\min_{P_g^t} \sum_{g \in G} f_g\left(P_g^t\right) \quad (12)$$

s.t.

$$\sum_{g \in G} P_g^t + W^t = D^t \quad (13)$$

$$P_{g,\min} \le P_g^t \le P_{g,\max},\ g \in G \quad (14)$$

Equation (12)-(14) form the economic dispatch model.

Step 1) Set the iteration counter $i=1$, $D_i^t = \sum_{g \in G} P_{g,\min}$.

Step 2) Solve the optimization problem (12)-(14) using linear programming or quadratic programming approaches. $P_s^t = \sum_{g \in G} P_g^t$.

Step 3) Calculate the equivalent ramp-up rate $\Re_{up}\left(P_s^{t-1}\right)$ and equivalent ramp-down rate $\Re_{dn}\left(P_s^{t-1}\right)$ using (10) and (11), respectively.

Step 4) While $D_i^t < \sum_{g \in G} P_{g,\max}$, set $i = i+1$, $D_i^t = D_i^t + d$, and go to Step 2). Otherwise, go to Step 5).

Step 5) Terminate the algorithm.

The ERRF can then be expressed using a series of the tuples $(\Re_{up}\left(P_s^{t-1}\right), P_s^{t-1})$ and $(\Re_{dn}\left(P_s^{t-1}\right), P_s^{t-1})$, which will be shown in Section IV.

Given a thermal power system, the above procedure can be conducted in an "offline" manner. The obtained ERRF can be utilized in various applications such as potential wind curtailment analysis.

IV. Case Studies

In this section, the IEEE 30-bus system [9] is used to demonstrate the effectiveness of the proposed ERRF model. CPLEX12.1 is used to solve MIP models [10]. Computer processor is Intel (R) Core (TM) i5-3210 @ 2.50GHz 2.50GHz. Installed memory is 4 GB.

A. Data

The topology of the IEEE 30-bus system is obtained from [9]. The wind farms are located at Bus 11 and 13. The ramp rates of thermal units are scaled down from [11]. The generator cost data are obtained from Matpower [12].

TABLE I
GENERATOR PARAMETERS FOR IEEE 30-BUS SYSTEM

Generating No.	Type	Max Cap(MW)	Min Power(MW)	Ramp Rate(MW/min)
1	Thermal	80	30	0.40
2	Thermal	80	30	0.40
3	Thermal	50	20	0.25
4	Thermal	50	20	0.25
5	Wind	30	0	/
6	Wind	30	0	/

TABLE II
GENERATOR COST FUNCTIONS FOR IEEE 30-BUS SYSTEM

Generating No.	Type	a	b
1	Thermal	0.02000	2.00
2	Thermal	0.01750	1.75
3	Thermal	0.06250	1.00
4	Thermal	0.00834	3.25

The summation of the minimum power of thermal units is 100 MW while that of the maximum power is 260 MW. The step-length d is set to be 0.5 MW in this case study. The network constraints are not considered.

B. Impact of cost functions on ERRF

To analyze the impact of different cost functions on ERRF, the linear and quadratic functions are utilized in this subsection.

The system-wide ramp-up and ramp-down rates with linear cost functions are shown in Figs. 1 and 2, respectively.

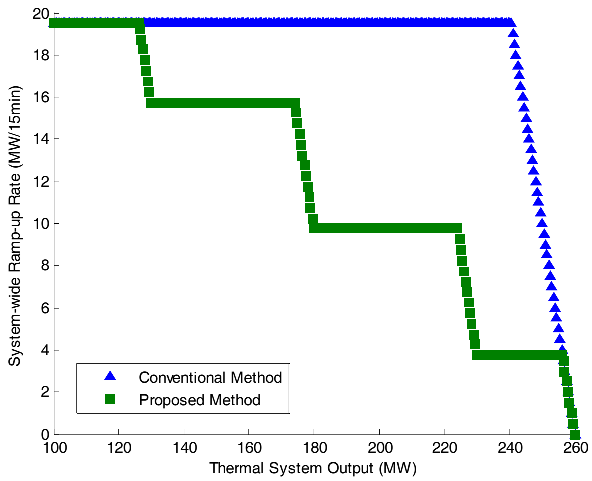

Fig. 1. System-wide ramp-up rate with linear cost functions

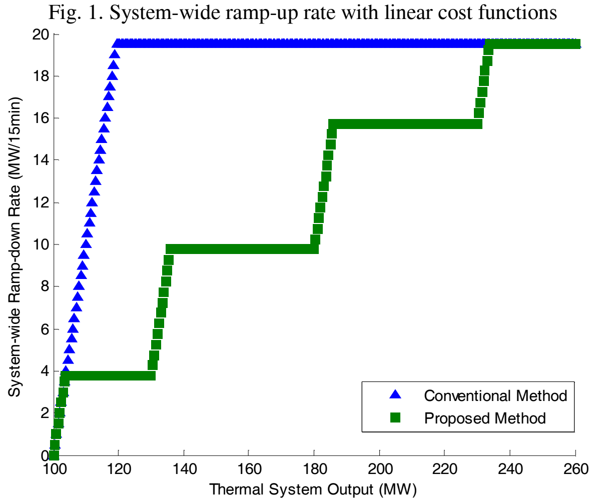

Fig. 2. System-wide ramp-down rate with linear cost functions

As one can observe, the proposed ERRF method produces lower system-wide ramping capability boundary. Given the principle of dispatch such as cost minimization, ERRF would shrink the feasible region of system operations.

When the cost function is linear, the system-wide ramping capability shows a staircase shape due to the characteristics of linear programming. The optimum is always achieved at the vertex.

The system-wide ramp-up and ramp-down rates with quadratic cost functions are shown in Fig. 3 and Fig. 4 respectively.

It is interesting to find that the system-wide ramp-up and ramp-down flexibility is not symmetric. As one can observe from Fig. 1 and Fig 3, the system-wide ramp-up flexibility is significantly different when the thermal system output is high, while it can be seen from Fig. 2 and Fig. 4, the system-wide ramp-down flexibility is significantly different when the thermal system output is low.

C. Potential wind curtailment analysis

Wind curtailment evaluation is taken as an example in this subsection to show one of the applications of the ERRF.

The potential wind curtailment using the conventional approach and the proposed ERRF approach are shown in Fig. 5 and Fig. 6, respectively.

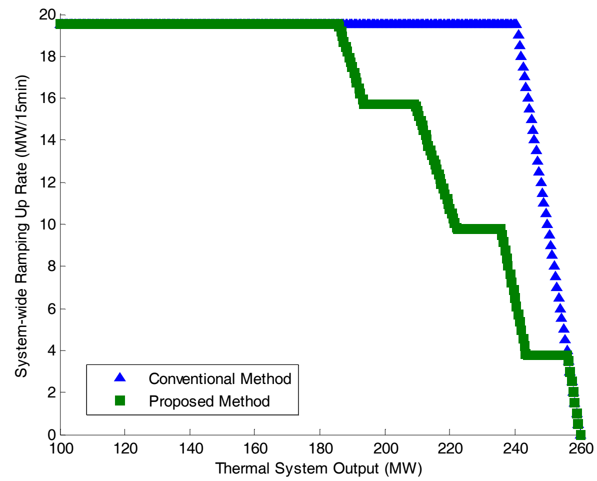

Fig. 3. System-wide ramp-up rate with quadratic cost functions

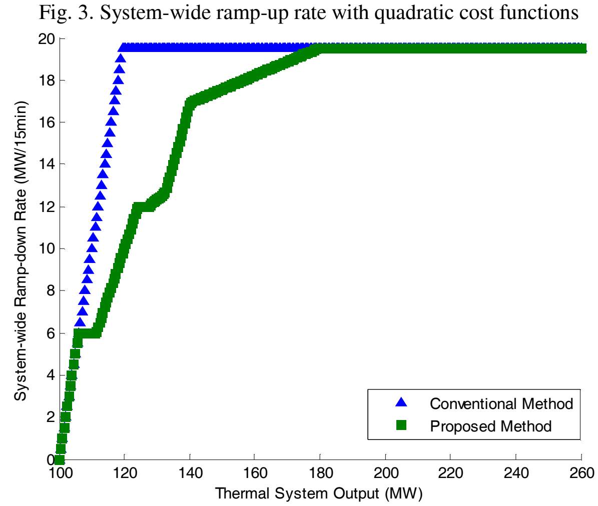

Fig. 4. System-wide ramp-down rate with quadratic cost functions

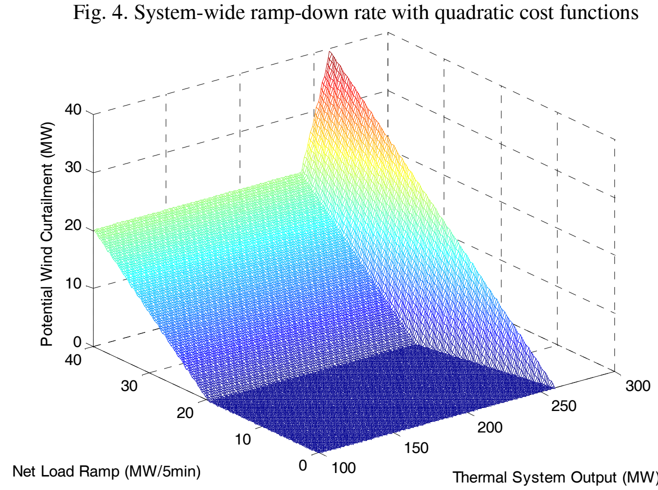

Fig. 5. Potential wind curtailment analysis using conventional approach

As one can observe from Fig. 5 and Fig. 6, with the same net load ramp, when the system load is higher, the potential wind curtailment is more due to less flexibility; and *vice versa*. With the same system load, when the net load ramp is bigger, the wind curtailment is more; and *vice versa*.

Given the principle of dispatch such as cost minimization, the impact of units' operating points on the system-wide ramp rate can be accurately considered in this model. It can be seen

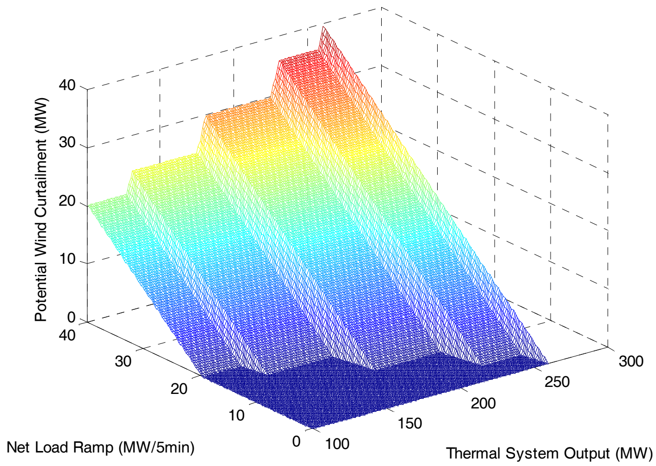

Fig. 6. Potential wind curtailment analysis using ERRF

from Fig. 5 and Fig. 6 that with conventional approach, the result tends to be *optimistic*. The proposed ERRF can reflect *more realistically* the system-wide ramping capability, and therefore, the wind curtailment would increase.

It is worth noting that since the ERRF is formulated using a single snapshot economic dispatch model, the inter-temporal constraints are not considered. Therefore, the look-ahead capability of thermal units are not reflected in ERRF. In a look-ahead economic dispatch framework, some generating units may ramp down in advance to prepare for the upcoming ramping hard time. Therefore, the wind curtailment can be less than the evaluation result with ERRF. The ERRF thus provides a simple method to assess the worst scenario of potential wind curtailment without running complex simulations.

This can be utilized as an "offline" evaluation tool for the system operator to be visually aware the potential wind curtailment under different system operational conditions.

V. Conclusions and Future Work

This paper proposes an equivalent ramp rate function (ERRF) for thermal power systems. Given the principle of dispatch such as cost minimization, the impact of the operating points of each thermal unit on the system-wide ramping flexibility is accurately considered. The impact of different cost functions on the shape of the ERRF is also analyzed in this paper. The case study based on the IEEE 30-bus system indicates that the proposed ERRF can reflect *more realistically* the system-wide ramping capability. The ERRF can be utilized as an "offline" evaluation tool to assess the potential wind curtailment due to lack of ramping flexibility. The proposed ramp rate function can be applied in wind integration study, hydro-thermal coordination, multi-area economic dispatch, and etc.

The economic dispatch model without network constraints is utilized in this paper to formulate the ERRF. To consider line capacity and inter-temporal constraints in the formulation procedure of ERRF would be very interesting, yet more challenging. How to incorporate the non-convex ERRF into an optimization framework such as wind integration study can be a topic of future study.

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